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HEAT AND MASS TRANSFER IN THE PROCESSES OF DRYING OF MATERIALS

CERTAIN REGULARITIES OF THE KINETICS OF DRYING OF MOIST MATERIALS

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A method for calculating the kinetics of drying under the conditions of continuous variation in the Rehbinder number in the period of decreasing drying rate is considered.

Such methods of calculation of the kinetics of drying that contain a minimum number of constants determined experimentally are of interest for the practice of drying. Methods based on the most generalized regularities of the process of drying [1, 2] are preferable.

In [3, 4], we have obtained equations enabling us to determine the temperature of material in the period of decreasing drying rate. However, when experimental data were generalized, the heat expended in heating the material (Rehbinder number) was calculated for each region of the process from the empirical formula, whereas in integrating the equation by the temperature curve, the Rehbinder number was taken as a step function. Under actual conditions, the dependence of the Rehbinder number on the moisture content is continuous.

Let is write the energy- and moisture-balance equation for the period of decreasing drying rate:

$$\bar{\alpha}\,(t_{\text{med}} - t_{\text{s}})\,F = rm_0\,\frac{d\bar{u}}{d\tau} + (c_0 m_0 + c_{\text{m}} m_{\text{m}})\,\frac{d\bar{t}}{d\tau}\,. \tag{1}$$

Here the left-hand side of the equation is the quantity of heat supplied to the entire surface of a body in the second period; the right-hand side is the sum of the quantity of heat for evaporation of moisture and heating of a moist material. Since the moisture content of the body is $\bar{u} = m_{\text{m}}/m_0$, we obtain $cm_0 = m_0(c_0 + c_{\text{m}}\bar{u})$, where $c = c_0 + c_{\text{m}}\bar{u}$ is the heat capacity of the moist material.

The heat-flux density in the first period [1, 5] is determined as

$$q_1 = \alpha_{\text{cr}}\,(t_{\text{med}} - t_{\text{w}}) = r\rho_0 R_V N\,. \tag{2}$$

We divide and multiply the left-hand side of Eq. (1) by (2) and use the relations establishing a link between the Nusselt number for the periods of decreasing and constant drying rate and the Rehbinder number [5] and obtained based on an extensive amount of experimental data:

$$\frac{\text{Nu}}{\text{Nu}_{\text{cr}}} = \frac{\bar{\alpha}}{\alpha_{\text{cr}}} = (1 + \text{Rb})\,N^{*^{0.57}}\,, \quad \frac{t_{\text{med}} - t_{\text{s}}}{t_{\text{med}} - t_{\text{w}}} = \Delta t^* = N^{*^{0.43}}\,.$$

After simple transformations, we have

$$Fr\rho_0 R_V N\,(1 + \text{Rb})\,N^* = rm_0\,\frac{d\bar{u}}{d\tau} + cm_0\,\frac{d\bar{t}}{d\tau}\,. \tag{3}$$

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TABLE 1. Values of the Constants A and n in Formula (9) for Certain Materials

Material	δ, mm	u_{eq}	u_{cr}	t_{med}, °C	φ, %	v, m/sec	A	n
Woolen felt	8—10	0.10	0.75	90—120	4—5	3—20	0.1	6.0
Sole leather	4.5	0.15	0.57	40—60	15	3—5	0.5	8.5
Felt	4	0.10	0.40	50	25—75	0.5—1	0.1	10
Nozzle-blown cloths	—	0.05	0.35	90—120	4—5	25	0.25	9.5

Determination of the Rehbinder number yields

$$\frac{d\bar{t}}{d\tau} = \frac{r}{c}\,\mathrm{Rb}\,\frac{d\bar{u}}{d\tau}\,. \tag{4}$$

With account for the equality $\rho_0 R_V = \frac{\rho_0 V_0}{F} = \frac{m_0}{F}$, Eq. (3) will take the form

$$\left|\frac{d\bar{u}}{d\tau}\right| = NN^{*}\,. \tag{5}$$

Thus, the drying rate at any instant of time of the second period is computed from the drying rate N in the first period and the relative rate N^{*}. The use of the Rehbinder number Rb and the relative drying rate N^{*} enables us to establish a link between heat exchange and moisture exchange, to calculate the intensity of heat exchange $q(\tau)$ from the intensity of moisture exchange N^{*}, and to avoid determination of the heat-exchange coefficients in the period of decreasing drying rate.

The equation of the temperature curve in the second period (4) with account for (5) will be written as

$$\frac{d\bar{t}}{d\tau} = \frac{r}{c}\,N\,\mathrm{Rb}\,N^{*}\,. \tag{6}$$

Consequently, knowing the regularities of variation in N^{*} and Rb with moisture content $\bar{u}$ with time τ, we can find the heat-exchange intensity and the material temperature in the second period of drying. From the method of generalization of drying curves [1, 2], it is seen that the generalized time $N\tau$ and the relative drying rate N^{*} are functions of the moisture content $\bar{u}$: $N^{*} = f_1(\bar{u})$, $N\tau = f_2(\bar{u})$, and hence $N^{*} = f(N\tau)$. An analysis of experimental data [2, 3] for different materials irrespective of the mode of energy supply has shown that the dependence $N^{*} = f(N\tau)$ is expediently sought in the form

$$N^{*} = \exp(-\,aN\tau)\,. \tag{7}$$

Processing of experimental data on drying of a large number of different capillary-porous materials [6] enabled us to establish a simple relation for computation of the quantity a in Eq. (7):

$$a = \frac{8\cdot 10^{-3}}{\bar{u}_{cr}}\,. \tag{8}$$

According to the data of different researchers [1, 2, 6], the value of the first critical moisture content $\bar{u}_{cr}$ is virtually independent of the regime parameters of the process of drying for many materials. The dependence of the Rehbinder number on the moisture content is determined by the empirical formula [1]

$$\mathrm{Rb} = A\exp\left(-\,n\,(\bar{u} - u_{eq})\right)\,. \tag{9}$$

Table 1 gives the values of the constants A and n for certain materials.

Using the existing relations for the heat of vaporization $r = 2500 - 2.3\bar{t}$ and the heat capacity of a moist body $c = c_0 + c_m\bar{u}$ and introducing the notation $r_0 = 2500$ kJ/kg and $r_1 = 2.3$ kJ/kg, we write $r = r_0 - r_1\bar{t}$. Differential equa-

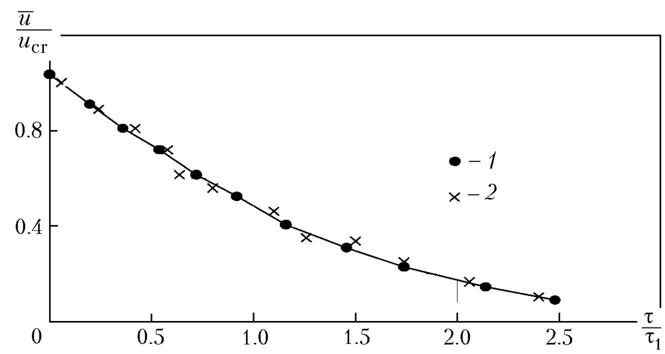

Fig. 1. Dimensionless curves of drying for sole leather: 1) experiment; 2) calculation from Eq. (14).

tions of the temperature curve $\bar{t}(\tau)$ and $t(\bar{u})$ for the period of decreasing drying rate with account for Eqs. (4)–(9) have the form

$$\frac{d\bar{t}}{d\tau} = -\frac{r_0 - r_1 t}{c_0 + c_m \bar{u}} NA \exp\left(-n(\bar{u} - u_{eq}) - aN\tau\right), \tag{10}$$

$$\frac{d\bar{t}}{d\bar{u}} = -\frac{r_0 - r_1 t}{c_0 + c_m \bar{u}} A \exp\left(-n(\bar{u} - u_{eq})\right). \tag{11}$$

Separating the variables and integrating (11), we obtain the equation of the temperature curve as a function of the moisture content u:

$$\bar{t}(\bar{u}) = \frac{r_0}{r_1} + \left(t_w - \frac{r_0}{r_1}\right) \exp\left(\frac{r_1}{c_m} \mathrm{Rb}_{cr} \exp\left(n\left(\frac{c_0}{c_m} + \bar{u}_{cr}\right)\right)\left(E_1\left(n\left(\frac{c_0}{c_m} + \bar{u}_{cr}\right)\right)\right) - E_1\left(n\left(\frac{c_0}{c_m} + \bar{u}\right)\right)\right), \tag{12}$$

where Rb_{cr} is determined from formula (9) for $\bar{u} = u_{cr}$.

The drying time in the period of decreasing rate is determined by integration of (5) with account for (8) on condition that $\frac{du}{d\tau} > 0$:

$$\tau = -\frac{1}{aN} \ln\left(1 - a(u_{cr} - \bar{u})\right). \tag{13}$$

Since the drying rate in the first period is $N = (u_0 - u_{cr})/\tau_1$, the equation of the drying curve will be written in dimensionless form:

$$\frac{\tau}{\tau_1} = -\frac{1}{a(u_0 - u_{cr})} \ln\left(1 - a u_{cr}\left(1 - \frac{\bar{u}}{u_{cr}}\right)\right). \tag{14}$$

Calculations from dependences (12) and (14) for different materials (porous ceramics, woolen felt, asbestos, felt, clay, and sole leather) were carried out on a computer with the approximation function $E_1(x)$ [7]:

$$E_1(x) = \int_x^\infty \exp(-t)\,\frac{dt}{t}. \tag{15}$$

The results of the calculations from Eqs. (12) and (14) for sole leather in convective drying under forced-convection conditions in the ranges of variation in temperature $t_{med} = 40$–60^{o}C and air velocity $v = 3$–5 m/sec and for a

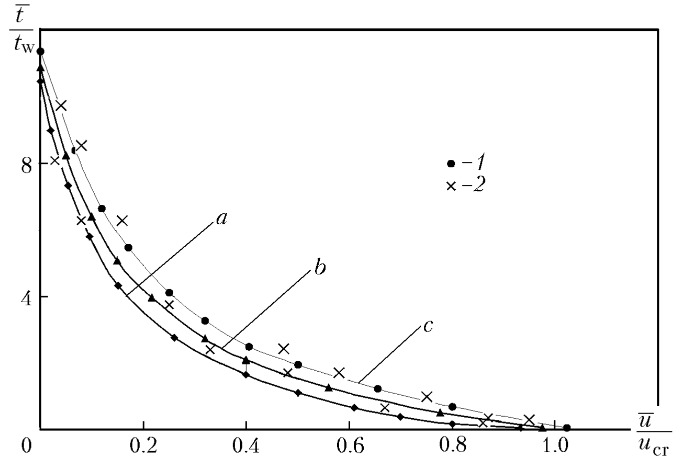

Fig. 2. Dimensionless temperature curves for sole leather: a) t_{med} = 40, b) 50, and c) 60°C; 1) experiment; 2) calculation from Eq. (12).

relative humidity φ = 15% and equilibrium moisture content of the leather $u_{eq} \approx 0.12$–0.15 are presented in Figs. 1 and 2 in the form of dimensionless dependences:

$$-\frac{\bar{u}}{u_{cr}} = f_1 \frac{\tau}{\tau_1}, \ \frac{\bar{t}}{t_w} = f_2 \frac{\bar{u}}{u_{cr}} \ .$$

Analogous dependences have been obtained for a large number of different capillary-porous materials (asbestos, woolen felt, cardboard, felt, cloth, and others).

A comparison of experimental data and those calculated from formulas (12) and (14) shows a satisfactory agreement for a large set of drying curves and temperature curves; their discrepancies of 5 to 8% are within the experimental accuracy.

Thus, analytical solution of the energy- and moisture-balance equation with the use of generalized regularities of the kinetics of drying, which are based on experimental data, has enabled us to obtain equations for determining the most important parameters of drying. Such an approach to investigating heat and moisture exchange brings drying theory closer to practice and makes it possible to find the most general empirical dependences. The results of investigations can be used in considering problems related to the modeling of dryers.

NOTATION

c, heat capacity of a moist material, kJ/(kg·°C); c_0 and c_m, heat capacities of a dry material and water, kJ/(kg·°C); F, area of the material surface, m^2; m_0 and m_m, mass of a perfectly dry material and water, kg; N, drying rate in the first period, 1/sec; N^*, relative drying rate; Nu and Nu$_{cr}$, Nusselt numbers in the period of decreasing and constant drying rate; q_1, heat-flux density in the first period, W/m^2; r, heat of vaporization, kJ/kg; Rb, Rehbinder number; R_V, ratio of the volume of a perfectly dry body to its surface area, m; t_{med} and t_s, temperatures of the medium and the material surface, °C; t_w, wet-bulb temperature, °C; v, air velocity, m/sec; u_0, u_{cr}, u, and u_{eq}, initial, critical, running, and equilibrium moisture content of the material; $\bar{u}$, moisture content of the body; V_0, volume of a perfectly dry body, m^3; $dt/d\tau$, rate of change in the material temperature; $du/d\tau$, drying rate in the second period; α and α_{cr}, heat-exchange coefficients for the period of decreasing and constant drying rate, W/(m^2·°C); δ, material thickness, mm; ρ_0, density of a perfectly dry body, kg/m^3; τ and τ_1, running times in the period of decreasing rate and in the first period, sec; φ, relative humidity of air, %. Subscripts: eq, equilibrium; med, medium; s, surface; m, moisture; w, wet; cr, critical; 0, initial state.

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